Mathematical Foundations for Joining Only Knowing and Common Knowledge

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Common Knowledge and Only-Knowing

Common knowledge: $C_G\varphi$ if each agent in G knows φ, and also knows that every agent in G knows φ, and knows that everyone in G knows that everyone in G knows φ, and so on. (e.g. muddy children)

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Only knowing: $O_A\varphi$ if the agent A knows φ ($K_A\varphi$) and moreover everything they know follows from φ.

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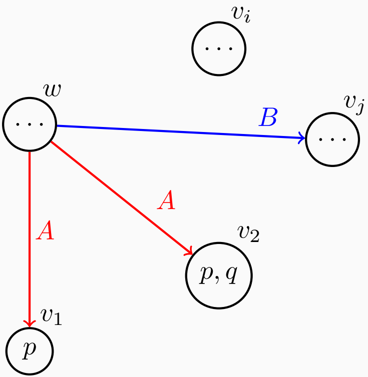

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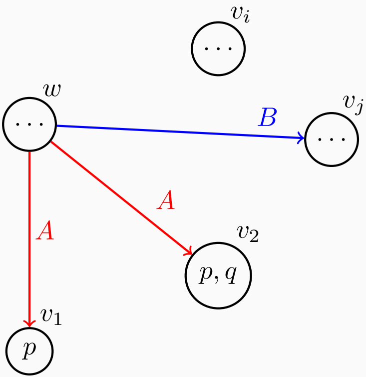

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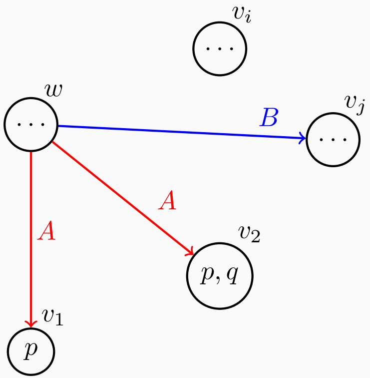

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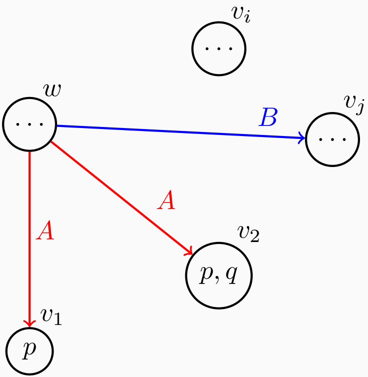

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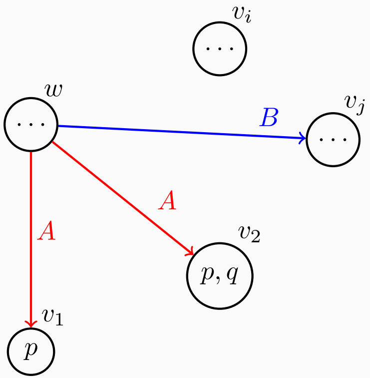

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What should the set $\mathcal{W}$ of worlds be?

A First Attempt

World — incorrect definition

Given a propositional vocabulary Σ, a *world* w consists of

- an interpretation w^{obj} over Σ, and
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$\longrightarrow$ Approximate the knowledge of agents up to a certain depth and define a $k+1$-world w as consisting, among others, of a set A^w of k-worlds for each agent A.

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Definition (completedness for successor ordinals)

A $\mu+1$-biworld w is *completed* if for all $A \in \mathcal{A}$, $A^w \cap \bar{A}^w = \emptyset$.

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Definition (World)

A *world* is a completed ω^2+1-biworld.

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9. (Fixed point axiom) $\models C_G\varphi \iff E_G(\varphi \wedge C_G\varphi)$.
10. (Induction rule) If $\varphi \models E_G(\varphi \wedge \psi)$, then $\varphi \models C_G\psi$.

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- Truthfulness makes $O_A(K_A p \vee K_A q)$ unsatisfiable, thus violating (O).

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Motivation for formally stating desirable properties of $\models$.

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Motivation for formally stating desirable properties of $\models$.

Main insight: Need $\bar{A}^w$

Problem with one-sided approximations: No criterion for completedness.

Conclusions

Goal: multi-agent epistemic logic with common knowledge and only knowing

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- Desirable properties of $\models$
- Studied adding truthfulness, positive introspection and negative introspection